7-6 Rational Zero Theorem

Solve, given that -4 is a zero.
$x^3 - 11x + 20 = 0$

$$\begin{array}{r|rrr} -4 & 1 & 0 & -11 & 20 \\ & & -4 & 16 & -20 \\ \hline & 1 & -4 & 5 & 0 \end{array}$$

$x^2 - 4x + 5 = 0$

$$x = \frac{4 \pm \sqrt{16 - 4(1)(5)}}{2}$$

$$\frac{4 \pm \sqrt{-4}}{2}$$

$$\frac{4 \pm 2i}{2}$$

$\{-4, 2 \pm i\}$

Rational Zero Theorem-- $\frac{p}{q}$ is a possible zero, where:

- p $\in$ set of integral factors of the constant
- q $\in$ set of the integral factors of the leading coefficient

Solve
$0 = 6x^3 + 7x^2 - 9x + 2$

$p \in \{\pm 1, \pm 2\}$

$q \in \{\pm 1, \pm 2, \pm 3, \pm 6\}$

$\frac{p}{q} \in \{\pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm 2, \pm\frac{2}{3}\}$

$$\begin{array}{r|rrrr} -2 & 6 & 7 & -9 & 2 \\ & & -12 & 10 & -2 \\ \hline & 6 & -5 & 1 & 0 \end{array}$$

$6x^2 - 5x + 1$

$$x = \frac{5 \pm \sqrt{25 - 4(6)1}}{2(6)}$$

$\frac{5 \pm 1}{12}$ $\frac{6}{12}$ $\frac{4}{12}$

$\{-2, \frac{1}{2}, \frac{1}{3}\}$

Solve
$x^4 - x^3 + 7x^2 - 9x - 18 = 0$

$p \in \{\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18\}$

$q \in \{\pm 1\}$

$\frac{p}{q} \in$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 7 & -9 & -18 \\ & & -1 & 2 & -9 & 18 \\ \hline \text{Depress } x^3 \to 2 & 1 & -2 & 9 & -18 & 0 \\ & & 2 & 0 & 18 & \\ \hline \text{Depress } x^2 \to & 1 & 0 & 9 & 0 & \end{array}$$

$x^2 + 9$

$\{-1, 2, \pm 3i\}$ $x = \pm 3i$

Do:
$0 = 2x^4 + 3x^3 + 6x^2 + 12x - 8$

Solve.
$0 = x^4 - 4x^3 + 6x^2 - 8x + 8$

$p \in \{\pm 1, \pm 2, \pm 4, \pm 8\}$
$q \in \{\pm 1\}$
$\frac{p}{q} \in$

$2 \mid 1 \; -4 \; 6 \; -8 \; 8$
$\quad\quad 2 \; -4 \; 4 \; -8$
$2 \mid 1 \; -2 \; 2 \; -4 \; | \; 0$
$\quad\quad 2 \; 0 \; 4$
$\quad 1 \; 0 \; 2 \; | \; 0$
$x^2 + 2$
$\{2, \pm i\sqrt{2}\}$ $x = \pm i\sqrt{2}$

Solve.
$0 = x^4 - 6x^3 + 8x^2 - 48x$ No constant
Factor a GCF
$x(x^3 - 6x^2 + 8x - 48) = 0$
$p \in$
$q \in$

HW
p381
13, 15, 19, 25, 28, 30

$\{0, 6, \pm 2i\sqrt{2}\}$